HA2_2008_End of Year Review Problems

Folks, here are some good practice problems for you to consider courtesy of Mr. Paul Foerster (your text book author).

3. Multiply and simplify.

a. $(\sqrt{8} + \sqrt{3})(\sqrt{15} + \sqrt{2})$

b. $(\sqrt{3} - \sqrt{5})^2$

c. $(\sqrt{y} - 6)(\sqrt{y} + 6)$

d. $(3x + 11)(2x - 7)$

e. $(x + 3)(x^2 - 4x - 5)$

4. Factor completely.

a. $a^5 + b^5$

b. $5x^2 + 27x + 36$

c. $x^3 + 2x^2 - 11x - 12$

d. $64p^2 - 36$

5. Carry out the indicated operations and simplify.

a. $\dfrac{1}{x^2 + 4x + 3} + \dfrac{1}{x^2 - 1}$

b. $\dfrac{9 - x^2}{3x + 9} \div \dfrac{x^2 - 5x + 6}{x^2 + 2x + 1}$

6. Simplify the following. *No* decimal approximations!

a. $5x^{-\frac{2}{5}} \cdot 7x^{\frac{3}{4}}$

b. $\sqrt{125} \div \sqrt[3]{25}$

c. $\dfrac{12}{\sqrt{19} + 4}$

d. $\dfrac{24}{\sqrt[3]{2}}$

e. $\sqrt[6]{36}$

f. $7\sqrt{45} + \dfrac{10}{\sqrt{5}}$

g. $\log_9 27$

h. $\log_7 30 + 2 \log_7 5$

i. $\left(\dfrac{5}{x-2} + 1\right) \div \left(\dfrac{5}{x+3} - 1\right)$

7. Solve the equation.

a. $3^{2x} = 457$

b. $2 + \sqrt{x-2} = x$

c. $x + \dfrac{1}{x-2} = \dfrac{x-3}{2-x}$

8. Given $f(x) = \dfrac{x^3 + 2x^2 - 11x - 12}{x^2 - x - 6}$,
tell the ordered pair at which there is a removable discontinuity.

1. Given $f(x) = x^3$ and $g(x) = 3^x$. One of the functions is an exponential function and the other is not. Which is which? How do you tell?

2. Evaluate: -7^4

3. Evaluate: $5 \cdot 8 + 2 \cdot 4^3$

4. Use variables to state the following:
   a. Definition of logarithm
   b. Property of a quotient of powers with equal bases.
   c. Property of the log of a product
   d. Definition of $x^{\frac{1}{n}}$.

5. Simplify: $(3r^5m^7)^4(64r^0m^{12})^{\frac{2}{3}}$

6. Evaluate $\sqrt[9]{32586.4}$

7. Evaluate in scientific notation. Round appropriately.

$$\frac{3.7495 \times 10^{-21}}{7.24 \times 10^{17}}$$

8. If $f(x) = 7^x$, write an equation expressing $f^{-1}(x)$ in terms of x.

9. Solve for x and simplify: $\log_{25}x = \frac{3}{2}$

10. Find $\log_6 37.9$.

11. Solve for x: $\log_x 64 = \frac{2}{3}$

12. Express as a single log of a single argument:

$$3 \log_5 12 - \log_5 36$$

Factor Completely:

1. $12a^3b^5 + 18ac$

2. $9x^2 - 36$

3. $x^2 + 14xy - 15y^2$

4. $4x^2 + 2x - 30$

5. $4z^2 + 25z + 6$

6. $9x^2 - 48x + 64$

7. $x^3 + c^3$

8. $8p^3 - f^3$

9. $(x + a)y + (x + a)(3p - 4)$

Long divide. Write the answer in mixed-number (or polynomial) form.

15. $\dfrac{4x^3 - 3x^2 + 7x - 19}{x - 2}$

16. $\dfrac{x^3 - 11}{x + 3}$

Sketch the graph, showing especially the behavior where the denominator equals zero.

17. $f(x) = \dfrac{1}{x - 3}$

18. $g(x) = \dfrac{(x + 2)(x - 3)}{x + 2}$

Simplify # 1-5

1. $\dfrac{(x+3)(x-5)}{(x-4)(x-1)} \div \dfrac{(x+3)(5-x)}{(x+4)(x+7)}$

2. $\dfrac{x}{x-2} - \dfrac{3}{x+4}$

3. $\dfrac{x^5+y^5}{x+y}$

4. $\dfrac{x+\dfrac{2}{x+3}}{2+\dfrac{x}{x+3}}$

5. Multiply: $(3x-7)(x^2-4x+5)$

6. Factor completely: x^3+125

7. Factor completely: $64x^2-16$

8. Given: $P(x) = x^3 - 5x^2 - 8x + 48$
   a. Show that $P(-3) = 0$.
   b. Factor $P(x)$ completely.

An "improper" algebraic fraction can be transformed by long division.

9. Write in mixed-number form:

$$\frac{3x^3 - 14x^2 - 29x + 5}{x-6}$$

Operation with rational functions sometimes involves solving fractional equations.

10. Solve: $x + \dfrac{1}{x-1} = \dfrac{x}{x-1}$

Write in simple *radical* form:

1. $\sqrt{18} + 2\sqrt{50} - \sqrt{98}$

2. Simplify: $\dfrac{21}{\sqrt{12} + \sqrt{5}}$

3. Simplify: $\dfrac{1 - \dfrac{1}{\sqrt{3}}}{1 + \dfrac{1}{\sqrt{3}}}$

Solving radical equations sometimes leads to extraneous solutions. Solve and check.

4. $\sqrt{x + 3} + \sqrt{x - 2} = 5$

Graphs of irrational functions show things about extraneous solutions. Let $f(x) = x - \sqrt{x - 2}$.

7. Find $f(11)$, $f(6)$, $f(3)$, $f(2.5)$, $f(2)$, and $f(0)$.

8. Plot the graph of f.

9. What is the domain of x if $f(x)$ is to be a *real* number?

10. Show that there are *no* real values of x for which $f(x) = 0$.

11. Set $f(x) = 4$, and solve for x. Show on your graph that one solution is valid, but the other one is extraneous.

1. For the arithmetic series $13 + 19 + \dots$,
   a. find t_{27}.
   b. find S_{27}.
   c. if $t_n = 445$, find n.

2. For the geometric series $50 + 48 + \dots$,
   a. 867.373 is the approximate value of a partial sum of the series. How many terms were added to get this number?
   b. To what number does the series converge?

3. Write four arithmetic means between 35 and 98.

4. a. Evaluate $\sum_{k=1}^{5} 2^k + 3$.
   b. Is this series arithmetic, geometric, or neither? Justify your answer.

1. Write the general equation that defines an exponential function.

2. Sketch the graph of the exponential function $f(x) = 2^x$ for values of x from -3 to 3.

3. Given the exponential function $f(x) = 3 \cdot 5^x$, evaluate $f(3)$, $f(-2)$, and $f(0)$.

To deal effectively with exponential functions you must know the properties of exponents. Complete the following.

4. $(ab)^k =$

5. $\frac{x^a}{x^b} =$

6. $(y^u)^p =$

7. The words "exponent," "base," and "power" apply to the symbol x^y. Tell which word goes with which part of the symbol.

Calculators can be used to raise numbers to non-integer powers. Evaluate the following.

8. $2^{3.7}$

9. $0.8^{-\frac{3}{7}}$

10. Explain why your answer to Problem 8 is *reasonable*.

Three definitions of exponentiation allow exponents to be zero, negative, or fractions. With these, you can operate on some powers *without* a calculator.

11. Use the definitions of exponentiation to show why $8^{-\frac{2}{3}} = 0.25$.

12. If $5^x = 625$, what does x equal?

13. Quick! What does 100^0 equal?

The properties of exponentiation let you transform expressions. Simplify the following.

14. $(4x^6y^{10})^3$

15. $\sqrt[6]{(11^{7.5})(11^{-5.1})}$

16. $(6x^{\frac{2}{3}})(5x^{-\frac{1}{3}}y^7)$

17. $\left(\frac{x^5}{y^6}\right)^7\left(\frac{y^8}{x^9}\right)^{10}$

18. $\frac{3 \cdot 2001^{501}}{5 \cdot 2001^{500}}$

7. Plot the graph of the rational function, showing all critical features.

$$f(x) = \frac{6x^2 + 12x - 90}{x^3 + 4x^2 - 11x - 30}$$

8. a. Write in simple radical form: $\frac{92}{\sqrt{3} + 7}$

11. a. Find t_{78} for the arithmetic sequence 3894, 3826, ...

b. Find S_{60} for the geometric series with first term 300 and common ratio 1.03.

c. Find the number to which the geometric series 100 + 90 + 81 + ... converges.